Slopefields

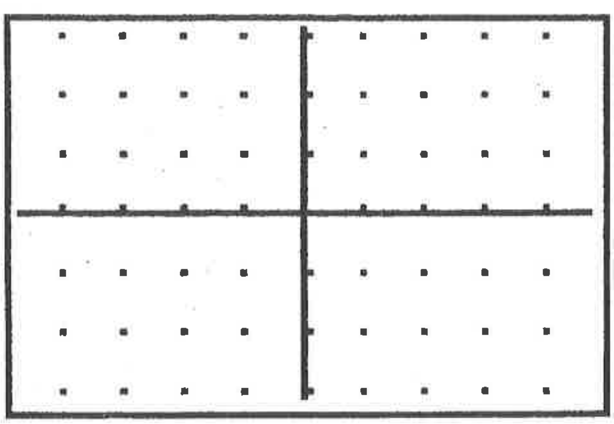
An equation like $\frac{dy}{dx} = y \ln x$, containing a derivative is a differential equation. The problem of finding a function y of x when we are given its derivative and its value at a particular point is called an **initial value problem**. The value of f, for one value of x, is the **initial condition** of the problem. When we find all the functions y that satisfy the differential equation we have **solved the differential equation**. When we then find the particular solution that fulfills the initial condition, we have solved the **initial value problem.** (FDWK)

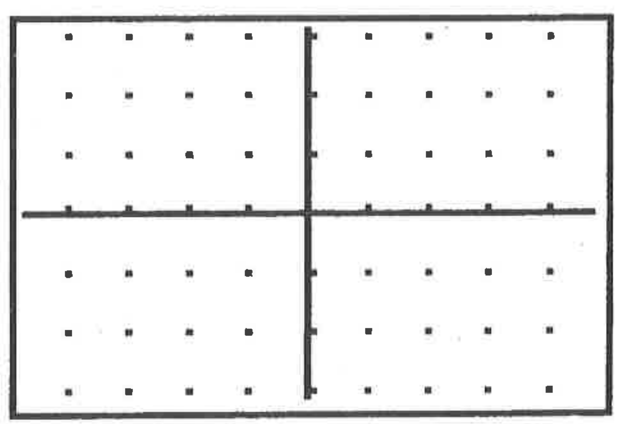
Slopefields and Euler's Method are used to understand and solve equations involving derivatives (i.e. differential equations) and are best understood as extensions of local linearity.

Students may best understand slopefields by drawing one themselves. Using a grid and the derivative of a well-known function, they can draw in small segments of tangent lines. This comes back to the idea that a differentiable function is "locally linear" and can be approximated by its tangent line in a small interval.

a. $\frac{dy}{dx} = 2x$

b. $\frac{dy}{dx} = \frac{1}{x+3}$





To obtain a particular solution with an initial condition, find that point on the slopefield and go in the direction of the tangent line. By connecting the small segments of tangent, we visually construct the solution to the initial value problem. This is another example of local linearity. (Broadwin)

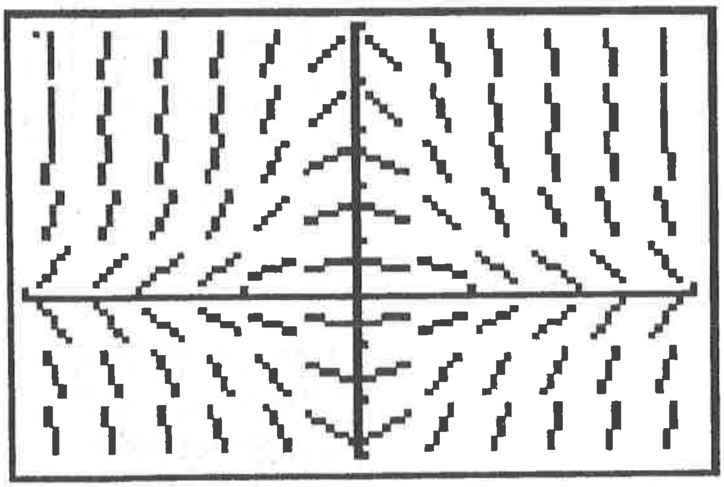
For graph a, assume the solution curve passes through (-2, -1). Sketch the solution curve

For graph b, assume the solution curve passes through (-2.9, -3). Sketch the solution curve

The slopefield for any first-order differential equation offers a simple and natural method of approximating solution curves. However, drawing a complete field of tangent segments by hand is a tedious task and is best left to a computer or graphing calculator. Typical slopefield programs take as input a differential equation $y' = f(x , y)$ where f could involve both x and y. The program computes the value of the derivative at several predetermined points (x , y) and then plots a short line segment centered at each point having the value of the derivative as slope. An appropriate viewing rectangle should be defined before executing the program. The program asks for the number of marks across and down; twelve across and eight down are good choices. Note that the function is stored in Y1. (BCC)

The particular solution to a differential equation depends on the particular initial condition given in the problem. Consider:

$$\frac{dy}{dx} = -2xy$$

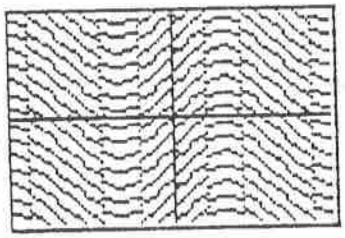


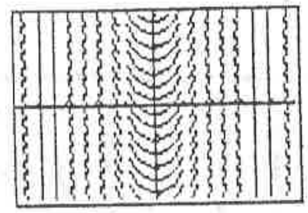
a) Sketch the solution curve through the point (0 , 1)

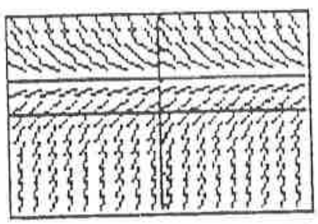
b) Sketch the solution curve through the point (0 , 2)

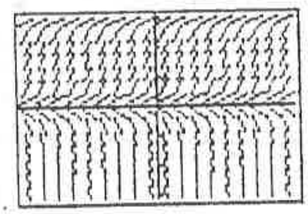
c) Sketch the solution curve through the point (0 , -2)

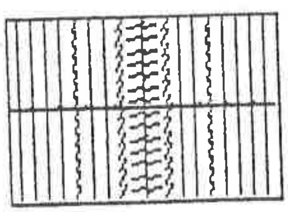
Below are 6 examples of SLOPEFIELDS. Match them with their differential equations.

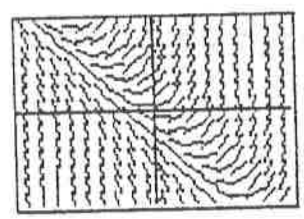
a. 

b. 

c. 

d. 

e. 

f. 

1. $\frac{dy}{dx} = 3x^2$ 2. $\frac{dy}{dx} = 1 - y$ 3. $\frac{dy}{dx} = \cos x$ 4. $\frac{dy}{dx} = x + y$ 5. $\frac{dy}{dx} = 2x$

6. $\frac{dy}{dx} = y(3 - y)$

All graphs are drawn in a ZOOM dec window. $-4.7 \le x \le 4.7$
$-3.1 \le y \le 3.1$